MMSE Symbol Level Precoding Under a Per Antenna Power Constraint for Multiuser MIMO Systems With PSK Modulation

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***Abstract*—This study proposes two MMSE symbol-level precoding approaches considering a strict per antenna power constraint and PSK modulation for perfect and imperfect channel state information scenarios. The proposed designs are formulated as second-order cone programs, allowing for an optimal solution via the interior point method. Numerical results confirm that, for the scenario with perfect channel state information, the proposed designs outperform the existing techniques in terms of bit-error-rate for low and intermediate signal-to-noise-ratio. Numerical evaluations also confirm the superiority of the proposed robust MMSE design in the presence of channel state information imperfection.**

***Index Terms*—MMSE, symbol-level precoding, strict per antenna power constraint, MIMO systems, second-order cone programming.**

I. Introduction

Massive multiuser multiple-input multiple-output (MU-MIMO) systems are considered as a promising technology and are expected to be vital for future wireless communications networks [1]. For MU-MIMO systems a fundamental problem is the design of low-complexity precoding algorithms that attain the high reliability constraints of future wireless communications networks.

Linear techniques such as zero forcing (ZF) and matched filtering [2], [3] are known to be asymptotically optimal [4] for massive MIMO systems due to the favorable propagation effects that rise for infinitely large arrays. However, when considering linear precoding, an established assumption in the literature [5], [6] is that the transmit symbols are constrained by an average total power constraint (TPC). This yields a system that is easier to model, yet, according to [7], in a realistic scenario each base station (BS) antenna is connected to its own power amplifier (PA) and thus has to meet its specific power constraints.

With this, several precoding techniques arose considering per antenna power constraints (PAPC). Linear channel-level precoding strategies considering an average PAPC are well studied in the literature [8], [9], [10], [11]. However, according to [12], the consideration of a strict PAPC (SPAPC) yields a more realistic scenario since the transmit power at each antenna is upper bounded by a threshold to avoid severe distortion at the PA due to clipping. With this, different linear precoding techniques have been developed considering SPAPCs [12], [13]. More recently, the symbol-level precoding (SLP) strategy has been receiving increasing attention since it allows for a higher degree of reliability. In [14] a SLP method is devised considering a per antenna transmit power minimization under the condition of attaining quality of service constraints for M-PSK modulations. In [15] SLP is considered with a SPAPC and two novel strategies based on the concept of *strict* and *non strict rotation* for constructive interference (CI) based precoding are proposed. SLP is considered also with other constraints, e.g., low-resolution [16], [17], [18] and constant envelope [19].

Besides the aforementioned concepts, one of the most prominent design criteria in the literature is the minimum mean squared error (MMSE). The MMSE utilization ranges from the established channel-level linear precoding strategy presented in [5] to a SLP design considering coarse quantization [20]. Although prominent in the literature to the best of the authors' knowledge the MMSE objective has not been considered for SLP under a SPAPC.

In this context, by considering a SPAPC this study proposes two SLP techniques for PSK modulation. While the first method utilizes the MMSE criterion considering perfect channel state information (CSI) at the transmitter, the second approach allows for imperfect CSI scenarios by exploiting knowledge about second order statistics of the CSI mismatch. The proposed approaches are formulated in the standard second-order cone programming (SOCPs) form and are readily solved with polynomial complexity using the interior points method (IPM). Numerical results indicate that the proposed MMSE methods are superior to the existing techniques in terms of BER for the low and medium SNR regime. Moreover, regarding CSI imperfection the proposed robust MMSE (RMMSE) design outperforms the examined state-of-the-art algorithms for all values of CSI mismatch.

The remainder of this paper is organized as follows: Section II describes the system model. Section III exposes the MMSE and RMMSE optimization problems, formulates them as SOCPs and provides the complexity analysis of the proposed algorithms. Section IV presents and discusses numerical results, while Section V gives the conclusions.

Regarding the notation, bold lower case and upper case letters indicate vectors and matrices, respectively. Non-bold letters express scalars. The operator $(\cdot)^T$ denotes transposition. S_+^n denotes the set of symmetric positive semidefinite matrices of dimension $n \times n$. The operator $R(\cdot)$ converts a complex-valued vector into the equivalent real-valued notation. For a given column vector $\boldsymbol{a} \in \mathbb{C}^M$ the equivalent real-valued

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vector $\boldsymbol{a}_{\mathrm{r}} = R(\boldsymbol{a})$ reads as

$$\boldsymbol{a}_{\mathrm{r}} = \left[\mathrm{Re}\left\{\boldsymbol{a}_1\right\} \ \mathrm{Im}\left\{\boldsymbol{a}_1\right\} \ \cdots \ \mathrm{Re}\left\{\boldsymbol{a}_M\right\} \ \mathrm{Im}\left\{\boldsymbol{a}_M\right\}\right]^T. \qquad (1)$$

For a given matrix $\boldsymbol{A} \in \mathbb{C}^{K\times M}$ the equivalent real-valued matrix $\boldsymbol{A}_{\mathrm{r}} = R(\boldsymbol{A})$ is given by

$$\boldsymbol{A}_{\mathrm{r}} = \qquad (2)$$
$$\begin{bmatrix} \mathrm{Re}\{a_{11}\} & -\mathrm{Im}\{a_{11}\} & \cdots & \mathrm{Re}\{a_{1M}\} & -\mathrm{Im}\{a_{1M}\} \\ \mathrm{Im}\{a_{11}\} & \mathrm{Re}\{a_{11}\} & \cdots & \mathrm{Im}\{a_{1M}\} & \mathrm{Re}\{a_{1M}\} \\ \vdots & & \ddots & & \vdots \\ \mathrm{Re}\{a_{K1}\} & -\mathrm{Im}\{a_{K1}\} & \cdots & \mathrm{Re}\{a_{KM}\} & -\mathrm{Im}\{a_{KM}\} \\ \mathrm{Im}\{a_{K1}\} & \mathrm{Re}\{a_{K1}\} & \cdots & \mathrm{Im}\{a_{KM}\} & \mathrm{Re}\{a_{KM}\} \end{bmatrix}.$$

The inverse operation is denoted as $C(\cdot)$ which converts equivalent real-valued notation into complex-valued notation.

II. System Model

The system model consists of a single-cell MU-MIMO scenario where the BS is equipped with M transmit antennas serving K single antenna users. A symbol level transmission is considered where s_k represents the data symbol to be delivered for the k-th user. Each symbol s_k is considered to belong to the set $\mathcal{S}$ that represents all possible symbols of a α_s-PSK modulation and is given by

$$\mathcal{S} = \left\{s : s = e^{\frac{j\pi(2i+1)}{\alpha_s}}, \text{ for } i = 1, \ldots, \alpha_s\right\}. \qquad (3)$$

The symbols of all users are described in a stacked vector notation as $\boldsymbol{s} = [s_1, \ldots, s_K]^T \in \mathcal{S}^K$. Based on $\boldsymbol{s}$ the precoder computes the transmit vector $\boldsymbol{x} = [x_1, \ldots, x_M]^T$. The entries of $\boldsymbol{x}$ are constrained by a SPAPC, meaning $|x_m|^2 \le \mathrm{P_A}$ for $m \in \{1, \ldots, M\}$, where $\mathrm{P_A}$ represents the maximum per antenna transmit power. A frequency flat fading channel described by the matrix $\boldsymbol{H} \in \mathbb{C}^{K\times M}$ is considered. The BS is considered to receive the CSI coefficients from the users which corresponds to the matrix $\tilde{\boldsymbol{H}} \in \mathbb{C}^{K\times M}$, which implies spatial correlation $\mathrm{E}\{\tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{H}}\} = K\boldsymbol{R}_{\mathrm{s}}$. It is considered that the spatial correlation matrix $\boldsymbol{R}_{\mathrm{s}}$ can be estimated and is known at the BS. It is considered that the spatial correlation is modeled by the Kronecker model [21], which implies that the entries of $\boldsymbol{R}_{\mathrm{s}}$ are in the form of $[r_{\mathrm{s}}]_{i,j} = \rho^{(i-j)^2}$ for $(i, j) \in \{1, \ldots, M\}^2$. The factor $\rho \in [0, 1]$ is the correlation index of neighboring antennas. In this study the channel model is described by

$$\boldsymbol{H} = \boldsymbol{N}\tilde{\boldsymbol{H}} + \sqrt{\boldsymbol{I} - \boldsymbol{N}^2}\boldsymbol{\Psi}\boldsymbol{R}_{\mathrm{s}}^{\frac{1}{2}}. \qquad (4)$$

The matrix $\boldsymbol{N} = \mathrm{diag}\,(\boldsymbol{\eta})$, with $\boldsymbol{\eta} = [\eta_1, \ldots, \eta_K] \in [0, 1]^K$ describes the user-specific quality of the CSI which can also be interpreted as the temporal correlation factor. It is considered that $\boldsymbol{N}$ can be estimated and is known at the BS. The matrix $\boldsymbol{\Psi}$, with $\boldsymbol{\psi}_k \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I})$ being the k-th row of $\boldsymbol{\Psi}$ for $k \in \{1, \ldots K\}$, describes the random part of the channel model. The received signal for all users $\boldsymbol{z}$ can be described as

$$\boldsymbol{z} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{w}, \qquad (5)$$

with its k-th entry z_k being the received signal from the k-th user. The vector $\boldsymbol{w} \sim \mathcal{CN}\left(0, \sigma_w^2\boldsymbol{I}\right)$ represents additive white Gaussian noise (AWGN). Each received symbol z_k is detected as $\hat{s}_k = D(z_k)$ where $\hat{s}_k$ denotes the detected symbol for the k-th user and $D(\cdot)$ the hard detection operation.

III. Proposed MMSE Precoding Designs under a Strict Per Antenna Power Constraint

In this section, we propose SLP designs based on the MMSE objective under a SPAPC for two different scenarios. In the first scenario perfect CSI is considered, meaning $\tilde{\boldsymbol{H}} = \boldsymbol{H}$. In the second scenario, it is considered that the BS has imperfect CSI and knowledge about the matrices $\tilde{\boldsymbol{H}}$, $\boldsymbol{N}$ and $\boldsymbol{R}_{\mathrm{s}}$. The MMSE objective, similar as proposed in [5], can be utilized under a SPAPC with the following problem

$$\min_{\boldsymbol{x},\beta} \mathrm{E}\left\{\|\beta\boldsymbol{z} - \boldsymbol{s}\|_2^2\right\} \qquad (6)$$
$$\text{s.t. } |x_m|^2 \le \mathrm{P_A}, \text{ for } m \in \{1, \ldots, M\}, \ \beta \ge 0.$$

Note that the real-valued factor β represents a theoretical automatic gain control which is part of the established MMSE objective as proposed in [5]. The factor β is computed by the precoder alongside the transmit vector $\boldsymbol{x}$. Yet, since in this study PSK modulation is considered, knowledge of β is not required for hard detection.

A. Proposed MMSE SPAPC Design

The MMSE optimization problem from (6) can be rewritten by substituting $\boldsymbol{z}$ in the objective which yields

$$\min_{\boldsymbol{x},\beta} \mathrm{E}\{\|\beta\boldsymbol{H}\boldsymbol{x} + \beta\boldsymbol{w} - \boldsymbol{s}\|_2^2\} \qquad (7)$$
$$\text{s.t. } |x_m|^2 \le \mathrm{P_A}, \text{ for } m \in \{1, \ldots, M\}, \ \beta \ge 0.$$

Note that $\boldsymbol{x}$ is a complex-valued vector and β a real-valued scaling factor. Since most established optimization algorithms consider real-valued variables, the problem from (7) is in the following rewritten in a real-valued notation which yields

$$\min_{\boldsymbol{x}_{\mathrm{r}},\beta} \mathrm{E}\{\|\beta\boldsymbol{H}_{\mathrm{r}}\,\boldsymbol{x}_{\mathrm{r}} + \beta\boldsymbol{w}_{\mathrm{r}} - \boldsymbol{s}_{\mathrm{r}}\|_2^2\} \qquad (8)$$
$$\text{s.t. } \left\{[\boldsymbol{x}_{\mathrm{r}}]_{2m-1}^2 + [\boldsymbol{x}_{\mathrm{r}}]_{2m}^2\right\} \le \mathrm{P_A}, \text{ for } m \in \{1, \ldots, M\}, \ \beta \ge 0,$$

where $\boldsymbol{w}_{\mathrm{r}} = R(\boldsymbol{w})$, $\boldsymbol{s}_{\mathrm{r}} = R(\boldsymbol{s})$ and $\boldsymbol{H}_{\mathrm{r}} = R(\boldsymbol{H})$, with the operator $R(\cdot)$ introduced in (1) and (2). Considering that perfect CSI is available at the BS, i.e., $\boldsymbol{H} = \tilde{\boldsymbol{H}}$, the problem from (8) can be expressed as an equivalent problem with

$$\min_{\boldsymbol{x}_{\mathrm{r}},\beta} \beta^2\boldsymbol{x}_{\mathrm{r}}^T\boldsymbol{H}_{\mathrm{r}}^T\boldsymbol{H}_{\mathrm{r}}\,\boldsymbol{x}_{\mathrm{r}} - 2\beta\boldsymbol{x}_{\mathrm{r}}^T\boldsymbol{H}_{\mathrm{r}}^T\boldsymbol{s}_{\mathrm{r}} + \beta^2K\sigma_w^2 \qquad (9)$$
$$\text{s.t. } \left\{[\boldsymbol{x}_{\mathrm{r}}]_{2m-1}^2 + [\boldsymbol{x}_{\mathrm{r}}]_{2m}^2\right\} \le \mathrm{P_A}, \text{ for } m \in \{1, \ldots, M\}, \ \beta \ge 0.$$

If $\beta \ge 0$ would be constant, the objective would be a convex quadratically constrained quadratic program (QCQP), since $\boldsymbol{H}_{\mathrm{r}}^T\boldsymbol{H}_{\mathrm{r}} \in S_+^{2M}$, [22, Sec. 4.4]. Yet, the objective is in general not jointly convex in β and $\boldsymbol{x}_{\mathrm{r}}$ [23, Appendix]. Nevertheless, it can be rewritten as an equivalent convex function by substituting the optimization variable $\boldsymbol{x}_{\mathrm{r}}$. In this context, we introduce a new optimization variable $\boldsymbol{x}_{\mathrm{s}} = \beta\boldsymbol{x}_{\mathrm{r}}$, similar as done in [20], [24]. With this, the optimization problem described in (9) can be rewritten as

$$\min_{\boldsymbol{x}_{\mathrm{s}},\beta} \boldsymbol{x}_{\mathrm{s}}^T\boldsymbol{H}_{\mathrm{r}}^T\boldsymbol{H}_{\mathrm{r}}\boldsymbol{x}_{\mathrm{s}} - 2\boldsymbol{x}_{\mathrm{s}}^T\boldsymbol{H}_{\mathrm{r}}^T\boldsymbol{s}_{\mathrm{r}} + \beta^2K\sigma_w^2 \qquad (10)$$
$$\text{s.t. } [\boldsymbol{x}_{\mathrm{s}}]_{2m-1}^2 + [\boldsymbol{x}_{\mathrm{s}}]_{2m}^2 \le \beta^2\mathrm{P_A}, \text{ for } m \in \{1, \ldots, M\}, \ \beta \ge 0.$$

The problem can be written in matrix form as

$$\begin{aligned}\min_{\boldsymbol{v}}\ & \boldsymbol{v}^T\boldsymbol{U}\boldsymbol{v}+\boldsymbol{p}^T\boldsymbol{v} \\ \text{s.t. } & \|\boldsymbol{E}_m\boldsymbol{v}\|_2 \le \boldsymbol{g}^T\boldsymbol{v}, \quad \text{for } m\in\{1,\ldots,M\}, \\ & \boldsymbol{a}^T\boldsymbol{v}\le 0\end{aligned} \tag{11}$$

where $\boldsymbol{v}=\left[\beta,\boldsymbol{x}_{\mathrm{s}}^T\right]^T$, $\boldsymbol{a}=\left[-1,\boldsymbol{0}^T\right]^T$, $\boldsymbol{g}=\left[\sqrt{\mathrm{P_A}},\boldsymbol{0}^T\right]^T$, $\boldsymbol{p}=\left[0,-2\boldsymbol{s}_{\mathrm{r}}^T\boldsymbol{H}_{\mathrm{r}}\right]^T$,

$$\boldsymbol{U}=\begin{bmatrix}K\sigma_w^2 & \boldsymbol{0}\\ \boldsymbol{0}^T & \boldsymbol{H}_{\mathrm{r}}^T\boldsymbol{H}_{\mathrm{r}}\end{bmatrix},\quad \boldsymbol{E}_m=\begin{bmatrix}0 & \boldsymbol{0}\\ \boldsymbol{0}^T & \mathrm{diag}\left(\boldsymbol{d}_m\right)\end{bmatrix}, \tag{12}$$

with $\boldsymbol{d}_m\in\mathbb{R}^{2M\times 1}$ being a vector of zeros with ones at entries $2m-1$ and $2m$. Note that, the problem described in (11) is convex. In what follows it transformed into a SOCP in standard form, which significantly facilitates implementation. By introducing the additional variable t, cf. [22, Sec. 4.1.3], the problem can be written with quadratic constraints as

$$\begin{aligned}\min_{t,\boldsymbol{v}}\ & \boldsymbol{p}^T\boldsymbol{v}+2t+1 \\ \text{s.t. } & \|\boldsymbol{E}_m\boldsymbol{v}\|_2 \le \boldsymbol{g}^T\boldsymbol{v}, \quad \text{for } m\in\{1,\ldots,M\}, \\ & \boldsymbol{v}^T\boldsymbol{U}\boldsymbol{v}\le 2t+1 \\ & \boldsymbol{a}^T\boldsymbol{v}\le 0.\end{aligned} \tag{13}$$

Note that, since $\boldsymbol{U}\in S_+^{2M+1}$, it can be written as $\boldsymbol{U}=\boldsymbol{L}^T\boldsymbol{L}$, with $\boldsymbol{L}=\boldsymbol{U}^{\frac{1}{2}}$. By substituting $\boldsymbol{U}=\boldsymbol{L}^T\boldsymbol{L}$ and adding t^2 at both sides of the quadratic constraint the problem is rewritten as

$$\begin{aligned}\min_{t,\boldsymbol{v}}\ & \boldsymbol{p}^T\boldsymbol{v}+2t \\ \text{s.t. } & \|\boldsymbol{E}_m\boldsymbol{v}\|_2 \le \boldsymbol{g}^T\boldsymbol{v}, \quad \text{for } m\in\{1,\ldots,M\}, \\ & \boldsymbol{u}^T\boldsymbol{L}^T\boldsymbol{L}\boldsymbol{u}+t^2\le (t+1)^2 \\ & \boldsymbol{a}^T\boldsymbol{v}\le 0.\end{aligned} \tag{14}$$

By using stacked vector notation in the form of the new optimization variable $\boldsymbol{u}=\left[\boldsymbol{v}^T,t\right]^T$ and taking the square root of the quadratic constraint the problem can be rewritten as

$$\begin{aligned}\min_{\boldsymbol{u}}\ & \boldsymbol{r}^T\boldsymbol{u} \\ \text{s.t. } & \|\boldsymbol{F}_m\boldsymbol{u}\|_2 \le \boldsymbol{l}^T\boldsymbol{u}, \quad \text{for } m\in\{1,\ldots,M\}, \\ & \|\boldsymbol{G}\boldsymbol{u}\|_2\le \boldsymbol{q}^T\boldsymbol{u}+1 \\ & \boldsymbol{o}^T\boldsymbol{u}\le 0,\end{aligned} \tag{15}$$

where $\boldsymbol{r}=\left[\boldsymbol{p}^T,2\right]^T$, $\boldsymbol{l}=\left[\boldsymbol{g}^T,0\right]^T$, $\boldsymbol{q}=\left[\boldsymbol{0}^T,1\right]^T$, $\boldsymbol{o}=\left[\boldsymbol{a}^T,0\right]^T$,

$$\boldsymbol{F}_m=\begin{bmatrix}\boldsymbol{E}_m & \boldsymbol{0}\\ \boldsymbol{0}^T & 0\end{bmatrix},\quad \boldsymbol{G}=\begin{bmatrix}\boldsymbol{L} & \boldsymbol{0}\\ \boldsymbol{0}^T & 1\end{bmatrix}, \tag{16}$$

The problem described in (15) is a SOCP, cf. [22, Sec. 4.4.2], and can be readily solved with IPM. The solution can be converted back to complex-valued notation by extracting $\boldsymbol{x}_{\mathrm{s}}$ and β from $\boldsymbol{u}_{\mathrm{opt}}$ and applying $\boldsymbol{x}=C\left(\frac{\boldsymbol{x}_{\mathrm{s}}}{\beta}\right)$.

B. Proposed Robust MMSE SPAPC Precoding Design

In this subsection, we propose an SLP design based on the MMSE objective under a SPAPC considering knowledge of $\tilde{\boldsymbol{H}}$, $\boldsymbol{N}$ and $\boldsymbol{R}_{\mathrm{s}}$. Such that the MMSE objective under imperfect CSI is written in real-valued notation the matrices $\tilde{\boldsymbol{H}}_{\mathrm{r}}=R(\tilde{\boldsymbol{H}})$, $\boldsymbol{N}_{\mathrm{r}}=R(\boldsymbol{N})$, $\boldsymbol{\Psi}_{\mathrm{r}}=R(\boldsymbol{\Psi})$ and $\boldsymbol{R}_{\mathrm{s,r}}=R(\boldsymbol{R}_{\mathrm{s}})$ are defined. With this, the real-valued channel matrix can be written as $\boldsymbol{H}_{\mathrm{r}}=\boldsymbol{N}_{\mathrm{r}}\tilde{\boldsymbol{H}}_{\mathrm{r}}+\sqrt{\boldsymbol{I}-\boldsymbol{N}_{\mathrm{r}}^2}\boldsymbol{\Psi}_{\mathrm{r}}\boldsymbol{R}_{\mathrm{s,r}}^{\frac{1}{2}}$. By substituting $\boldsymbol{H}_{\mathrm{r}}$ in (6) and considering $\mathrm{E}\left\{\tilde{\boldsymbol{H}}_{\mathrm{r}}^T\boldsymbol{\Psi}_{\mathrm{r}}\right\}=\boldsymbol{0}$ the RMMSE problem reads as

$$\begin{aligned}\min_{\boldsymbol{x}_{\mathrm{r}},\beta}\ & \beta^2\boldsymbol{x}_{\mathrm{r}}^T\left(\tilde{\boldsymbol{H}}_{\mathrm{r}}^T\boldsymbol{N}_{\mathrm{r}}^2\tilde{\boldsymbol{H}}_{\mathrm{r}}+\gamma\boldsymbol{R}_{\mathrm{s,r}}\right)\boldsymbol{x}_{\mathrm{r}}-2\beta\boldsymbol{x}_{\mathrm{r}}^T\tilde{\boldsymbol{H}}_{\mathrm{r}}^T\boldsymbol{N}_{\mathrm{r}}\boldsymbol{s}_{\mathrm{r}}+\beta^2K\sigma_w^2 \\ \text{s.t. } & \left\{[\boldsymbol{x}_{\mathrm{r}}]_{2m-1}^2+[\boldsymbol{x}_{\mathrm{r}}]_{2m}^2\right\}\le \mathrm{P_A}, \ \text{for } m\in\{1,\ldots,M\},\ \beta\ge 0,\end{aligned}$$

where $\gamma=\mathrm{trace}(\boldsymbol{I}-\boldsymbol{N}_{\mathrm{r}}^2)$. As before, this proposed objective is not jointly convex in $\boldsymbol{x}_{\mathrm{r}}$ and β. Yet, an equivalent convex problem can be cast by substituting $\boldsymbol{x}_{\mathrm{s}}=\beta\boldsymbol{x}_{\mathrm{r}}$, which yields

$$\begin{aligned}\min_{\boldsymbol{x}_{\mathrm{s}},\beta}\ & \boldsymbol{x}_{\mathrm{s}}^T\left(\tilde{\boldsymbol{H}}_{\mathrm{r}}^T\boldsymbol{N}_{\mathrm{r}}^2\tilde{\boldsymbol{H}}_{\mathrm{r}}+\gamma\boldsymbol{R}_{\mathrm{s,r}}\right)\boldsymbol{x}_{\mathrm{s}}-2\boldsymbol{x}_{\mathrm{s}}^T\tilde{\boldsymbol{H}}_{\mathrm{r}}^T\boldsymbol{N}_{\mathrm{r}}\boldsymbol{s}_{\mathrm{r}}+\beta^2K\sigma_w^2 \\ \text{s.t. } & [\boldsymbol{x}_{\mathrm{s}}]_{2m-1}^2+[\boldsymbol{x}_{\mathrm{s}}]_{2m}^2\le \beta^2\mathrm{P_A}, \ \text{for } m\in\{1,\ldots,M\},\ \beta\ge 0.\end{aligned}$$

The problem can be written in matrix form as

$$\begin{aligned}\min_{\boldsymbol{v}}\ & \boldsymbol{v}^T\tilde{\boldsymbol{U}}\boldsymbol{v}+\tilde{\boldsymbol{p}}^T\boldsymbol{v} \\ \text{s.t. } & \|\boldsymbol{E}_m\boldsymbol{v}\|_2 \le \boldsymbol{g}^T\boldsymbol{v}, \quad \text{for } m\in\{1,\ldots,M\}, \\ & \boldsymbol{a}^T\boldsymbol{v}\le 0\end{aligned} \tag{17}$$

where

$$\tilde{\boldsymbol{U}}=\begin{bmatrix}K\sigma_w^2 & \boldsymbol{0}\\ \boldsymbol{0}^T & \tilde{\boldsymbol{H}}_{\mathrm{r}}^T\boldsymbol{N}_{\mathrm{r}}^2\tilde{\boldsymbol{H}}_{\mathrm{r}}+\gamma\boldsymbol{R}_{\mathrm{s,r}}\end{bmatrix},\quad \tilde{\boldsymbol{p}}=\begin{bmatrix}0\\ -2\tilde{\boldsymbol{H}}_{\mathrm{r}}^T\boldsymbol{N}_{\mathrm{r}}\boldsymbol{s}_{\mathrm{r}}\end{bmatrix},$$

and the other quantities are defined in (12). Note that, since $\tilde{\boldsymbol{U}}\in S_+^{2M+1}$ the problem is convex. By following the same steps utilized in the section III-A one can write the problem described in (17) as the following SOCP

$$\begin{aligned}\min_{\boldsymbol{u}}\ & \tilde{\boldsymbol{r}}^T\boldsymbol{u} \\ \text{s.t. } & \|\boldsymbol{F}_m\boldsymbol{u}\|_2 \le \boldsymbol{l}^T\boldsymbol{u}, \quad \text{for } m\in\{1,\ldots,M\}, \\ & \left\|\tilde{\boldsymbol{G}}\boldsymbol{u}\right\|_2\le \boldsymbol{q}^T\boldsymbol{u}+1 \\ & \boldsymbol{o}^T\boldsymbol{u}\le 0,\end{aligned} \tag{18}$$

where

$$\tilde{\boldsymbol{r}}=\begin{bmatrix}\tilde{\boldsymbol{p}}\\ 2\end{bmatrix},\quad \tilde{\boldsymbol{G}}=\begin{bmatrix}\tilde{\boldsymbol{L}} & \boldsymbol{0}\\ \boldsymbol{0}^T & 1\end{bmatrix},\quad \tilde{\boldsymbol{L}}=\tilde{\boldsymbol{U}}^{\frac{1}{2}} \tag{19}$$

and the other quantities are defined in (16). As before the problem described in (18) is a SOCP, cf. [22, Sec. 4.4.2], and can be readily solved with IPM. The solution can be converted back to complex-valued notation by extracting $\boldsymbol{x}_{\mathrm{s}}$ and β from $\boldsymbol{u}_{\mathrm{opt}}$ and applying $\boldsymbol{x}=C\left(\frac{\boldsymbol{x}_{\mathrm{s}}}{\beta}\right)$. Note that, the MSE associated to the solution of (17) is lower bounded by $\check{\mathrm{MSE}}\left(\boldsymbol{s}_{\mathrm{r}}\right)=K-\boldsymbol{s}_{\mathrm{r}}^T\boldsymbol{N}_{\mathrm{r}}^T\tilde{\boldsymbol{H}}_{\mathrm{r}}\left(\tilde{\boldsymbol{H}}_{\mathrm{r}}^T\boldsymbol{N}_{\mathrm{r}}^2\tilde{\boldsymbol{H}}_{\mathrm{r}}+\gamma\boldsymbol{R}_{\mathrm{s,r}}\right)^{-1}\tilde{\boldsymbol{H}}_{\mathrm{r}}^T\boldsymbol{N}_{\mathrm{r}}\boldsymbol{s}_{\mathrm{r}}$. This MSE bound, which is greater than zero due to the CSI imperfection, is computed by considering the unconstrained version of (17).

C. About the Complexity of the Proposed Designs

As mentioned the MMSE and the RMMSE optimization problems are SOCPs and thus can be solved via IPM. According to [25], the number of iterations of the primal-dual IPM can be upper bounded by $\sqrt{n}\log\left(n/\epsilon_{\mathrm{tol}}\right)$ where n is the number of variables and ϵ_{tol} is the predefined optimality tolerance.

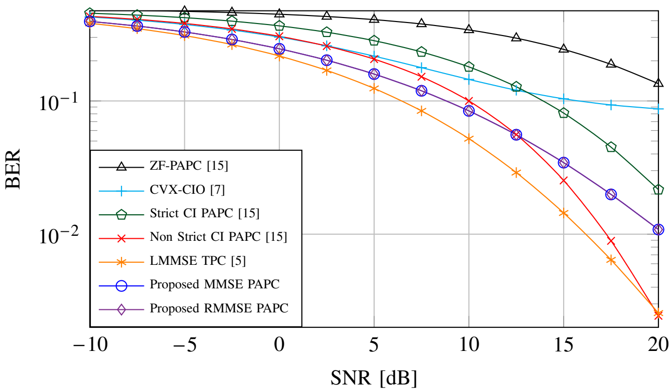

Fig. 1: BER $\times$ SNR for $K = 15$, $M = 15$, $\alpha_s = 4$, $\boldsymbol{\eta} = \mathbf{1}$, $\rho = 0$

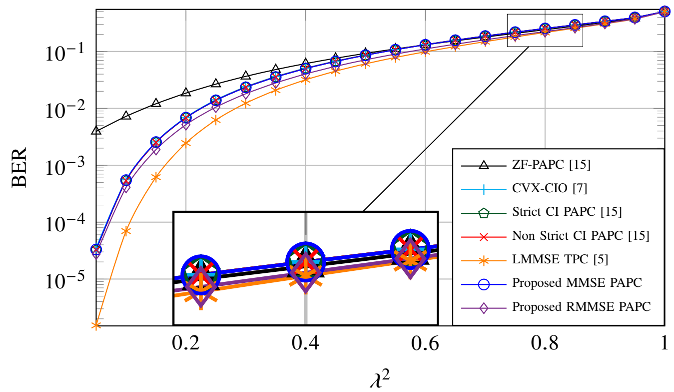

Fig. 2: BER $\times$ λ^2 for $K = 5$, $M = 50$, $\alpha_s = 8$, $\rho = 0$ and SNR= 12 dB

Note that, the complexity of the iterations is dominated by solving a linear system needed to compute the primal-dual search direction. With this, considering that the linear systems can be solved with complexity $O\left(n^3\right)$ via Gauss-Jordan elimination, the total complexity of the proposed approaches can be upper bounded by $O\left(M^{3.5}\log\left(M/\epsilon_{\text{tol}}\right)\right)$.

IV. Numerical Results

In this section, the proposed precoders are evaluated in terms of BER and computational complexity and compared with other state-of-the-art designs. To this end, the SNR is defined as SNR $= (M \cdot \mathrm{P_A})/\sigma_w^2$. The proposed methods are evaluated against the following state-of-the-art approaches: 1- The ZF SPAPC precoder [15]; 2- The CVX-CIO precoder [7] designed for constant envelope; 3- The Strict CI SPAPC precoder [15]; 4- The Non-Strict CI SPAPC precoder [15] and 5- The LMMSE precoder [5] (average TPC).

A. BER evaluation under perfect CSI

In this subsection, a BER $\times$ SNR evaluation is considered assuming no spatial correlation, i.e., $\rho = 0$ and perfect CSI. In this context, the considered MIMO scenario consists of a BS with $M = 15$ antennas serving $K = 15$ users with QPSK user symbols, meaning that $\alpha_s = 4$. As can be seen in Fig. 1, the proposed methods outperform the existing approaches in terms of BER for the low and intermediate SNR regimes. For high-SNR, the proposed MMSE precoders outperform all investigated approaches except for the Non-Strict CI-based precoder [15]. This is expected since it is known that CI is nearly optimal for high SNR [26] and the MMSE criterion is favorable for low and medium SNR [20].

B. BER evaluation under imperfect CSI

In this subsection, the proposed approaches are evaluated in terms of BER with CSI imperfection. The evaluated MIMO scenario consists of a BS with $M = 50$ antennas which serves $K = 5$ users with $\alpha_s = 8$. To facilitate the analysis during this subsection it is considered $\boldsymbol{\eta} = \xi\mathbf{1}$, meaning that all channels have the same CSI quality. The CSI imperfection is then expressed in terms of $\lambda^2 = \sqrt{1 - \xi^2}$.

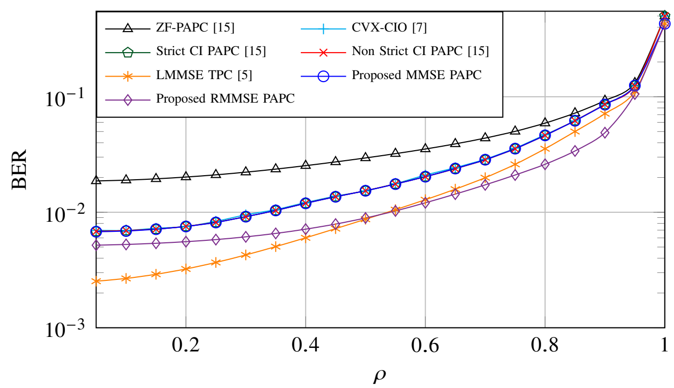

Fig. 3: BER $\times$ ρ for $K = 5$, $M = 50$, $\alpha_s = 8$, $\lambda^2 = 0.2$ and SNR= 12 dB

The first experiment consists of a BER performance evaluation for different levels of CSI imperfection under an SNR of 12 dB. For this experiment no spatial correlation is considered, meaning $\rho = 0$. As can be seen in Fig. 2 the proposed RMMSE SPAPC design outperforms in terms of BER all other examined SPAPC state-of-the-art approaches for $\lambda^2 > 0$. Moreover, the proposed RMMSE approach yields similar performance in terms of BER as the LMMSE [5] (average TPC) design for very low CSI quality ($\lambda^2 > 0.8$).

The second experiment consists of a BER performance evaluation against the spatial correlation factor ρ for SNR = 12 dB and $\lambda^2 = 0.2$. As shown in Fig. 3, the proposed RMMSE approach outperforms in terms of BER all examined SPAPC designs for all examined ρ. Moreover, it also outperforms the LMMSE design for $\rho > 0.5$.

Finally, the third experiment consists of a BER $\times$ SNR evaluation considering both imperfect CSI and spatial correlation with the parameters $\lambda^2 = 0.2$ and $\rho = 0.15$. As can be seen in Fig. 4, the proposed RMMSE precoder outperforms all other SPAPC approaches in terms of BER. Note that, both proposed MMSE and RMMSE approaches yield similar performance for low SNR. Starting from medium SNR, as the SNR grows, the proposed RMMSE approach deviates in performance from the proposed MMSE counterpart. Finally, for very high SNR (SNR > 27.5 dB) the proposed RMMSE approach shows significant advantage and outperforms also the LMMSE method (average TPC).

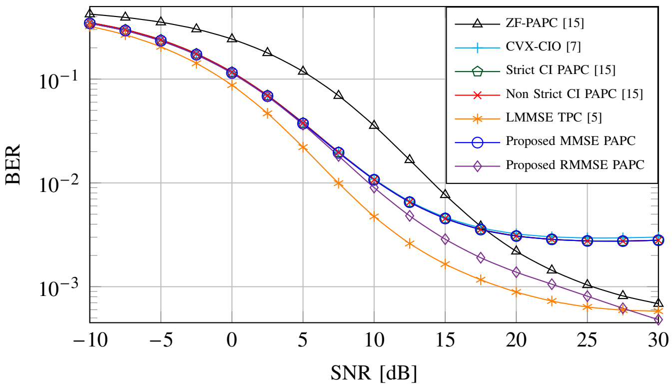

Fig. 4: BER × SNR for $K = 5$, $M = 50$, $\alpha_s = 8$, $\rho = 0.15$ and $\lambda^2 = 0.2$

C. Complexity Analysis

As discussed in section III-C the complexity of the proposed methods is upper bounded by $\mathcal{O}\left(M^{3.5}\log\left({}^{M}/_{\epsilon_{\text{tol}}}\right)\right)$. Table I summarizes the complexity of the considered approaches.

TABLE I: Computational Complexity of the Precoding Algorithms

Algorithm	Complexity
ZF SPAPC [15]	$\mathcal{O}\left(K^2 M\right)$
CVX-CIO [7]	$\mathcal{O}\left(M^{3.5}\log\left({}^{M}/_{\epsilon_{\text{tol}}}\right)\right)$
Strict CI SPAPC [15]	$\mathcal{O}\left(M^{3.5}\log\left({}^{M}/_{\epsilon_{\text{tol}}}\right)\right)$
Non-Strict CI SPAPC [15]	$\mathcal{O}\left(M^{3.5}\log\left({}^{M}/_{\epsilon_{\text{tol}}}\right)\right)$
Linear MMSE TPC [5]	$\mathcal{O}\left(K^3\right)$
Proposed MMSE SPAPC	$\mathcal{O}\left(M^{3.5}\log\left({}^{M}/_{\epsilon_{\text{tol}}}\right)\right)$
Proposed RMMSE SPAPC	$\mathcal{O}\left(M^{3.5}\log\left({}^{M}/_{\epsilon_{\text{tol}}}\right)\right)$

Note that, the optimization based state-of-the-art algorithms (namely CVX-CIO [7], Strict CI SPAPC [15] and Non-Strict CI SPAPC [15]) can be transformed in standard form SOCPs which can be solved via the primal-dual IPM with complexity $\mathcal{O}\left(M^{3.5}\log\left({}^{M}/_{\epsilon_{\text{tol}}}\right)\right)$. With this, it can be concluded that these approaches yield similar complexity as the proposed methods.

V. Conclusions

This study proposes two symbol-level precoding approaches considering a SPAPC and PSK modulation for perfect and imperfect CSI. The proposed precoding designs are formulated as SOCPs and are solved using the IPM in polynomial time. Numerical results confirm that for the perfect CSI scenario the proposed designs are superior to the existing techniques in terms of BER for low and intermediate SNR. Moreover, when considering imperfect CSI numerical evaluations underline the superiority of the proposed robust MMSE design.

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