ROBUST ADAPTIVE BEAMFORMING BASED ON POWER METHOD PROCESSING AND SPATIAL SPECTRUM MATCHING

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ABSTRACT

Robust adaptive beamforming (RAB) based on interference-plus-noise covariance (INC) matrix reconstruction can experience performance degradation when model mismatch errors exist, particularly when the input signal-to-noise ratio (SNR) is large. In this work, we devise an efficient RAB technique for dealing with covariance matrix reconstruction issues. The proposed method involves INC matrix reconstruction using an idea in which the power and the steering vector of the interferences are estimated based on the power method. Furthermore, spatial match processing is computed to reconstruct the desired signal-plus-noise covariance matrix. Then, the noise components are excluded to retain the desired signal (DS) covariance matrix. A key feature of the proposed technique is to avoid eigenvalue decomposition of the INC matrix to obtain the dominant power of the interference-plus-noise region. Moreover, the INC reconstruction is carried out according to the definition of the theoretical INC matrix. Simulation results are shown and discussed to verify the effectiveness of the proposed method against existing approaches.

***Index Terms*—** Adaptive beamforming, Covariance matrix reconstruction, Power method, Spatial spectrum match processing.

1. INTRODUCTION

Adaptive beamforming methods have been applied in wireless communications, sonar, and radar due to their interference mitigation capability [1]. However, under non-ideal conditions such as finite data samples and mismatches between the presumed and true steering vector (SV) the performance of adaptive beamformers degrades substantially. Several robust adaptive beamforming (RAB) techniques have been proposed to enhance robustness against the aforementioned mismatches, such as the linearly constrained minimum variance (LCMV) beamformer [2], diagonal loading (DL) [3], the eigenspace-based beamformer [4], the worst case-based technique [5], the probabilistically constrained approach in [6] and the modified robust Capon beamformer in [7]. Hence, the development of low-complexity RAB approaches has been a very active research topic in recent years. Nevertheless, a major cause of performance degradation in adaptive beamforming is the presence of the desired signal (DS) component in the training data, especially at high SNR.

To address this issue, many works tried to remove the signal-of-interest (SOI) components by reconstruction of the interference-plus-noise covariance (INC) matrix instead of using the sample covariance matrix (SCM). In [8], the INC matrix is reconstructed by integrating the nominal SV and the corresponding Capon spectrum over the entire angular sector except the region near the SOI. Several categories of INC matrix-based beamformers were then proposed, such as the beamformer in [9], which relies on a correlation coefficient method, the computationally efficient algorithms via low complexity reconstruction in [10], the sparse based method in [11], the subspace-based algorithm in [12], an approach based on spatial power spectrum sampling (SPSS) [13], and a method based on coprime array in [14]. The beamformer in [15] proposes a method in which each interference SV is estimated by the vector lying within the intersection of two subspaces while the algorithm in [16] constructs an INC matrix from the signal-interference subspace. The beamformer in [17] exploits orthogonal subspaces to eliminate the component of the SOI from the angle-related bases. The method in [18] uses the orthogonality of subspaces to reconstruct the INC matrix while in [19] a robust beamformer is proposed based on the principle of maximum entropy power spectrum (MEPS) to reconstruct the INC and the DS covariance matrices. Recently, an adaptive beamforming based on the idea of reconstructing the autocorrelation sequence of a random process from a set of measured data was reported in [20].

In this paper, we develop an effective RAB approach based on power method processing and spatial spectrum matching (PMP-SSM) to reconstruct the INC matrix, which aims to reconstruct more precisely the INC and DS matrices. The essence of the idea is that the power and the SV of the desired signal and of the interferences are estimated by the eigenvalues and eigenvectors lying respectively within the interval of the SOI and the interference angular regions. In order to accomplish this, we devise a simple approach based on the power method [21] where a simple iteration strategy is utilized for computing the dominant eigenvalues and corresponding eigenvectors. A key aspect of the proposed technique is to avoid the eigenvalue decomposition (EVD) required to reconstruct the INC matrix. In addition, an effective processing based on matching spectrum is developed to reconstruct the DS covariance matrix and a new SV estimation of SOI is obtained over the SOI angular sector using the estimated covariance matrix with the noise components excluded. In the proposed SV estimation algorithm, little prior information such as the imprecise knowledge of the antenna array geometry and the angular sectors is required, and the knowledge of the presumed steering vector is not essential.

2. PROBLEM BACKGROUND

Let us consider a uniform linear array (ULA) composed of M omnidirectional array elements. Assume that L narrowband signals (one SOI and $L-1$ interferences) impinge on the array from the directions $\{\theta_i\}_{l=1}^{L}$. The array received vector at time instant k, denoted by $\mathbf{x}(k) = \mathbf{x}_{\mathrm{s}}(k) + \mathbf{x}_l(k) + \mathbf{x}_{\mathrm{n}}(k)$, can be modeled as

$$\mathbf{x}(k) = s_1(k)\mathbf{a}_1 + \sum_{l=2}^{L} s_l(k)\mathbf{a}_l + \mathbf{x}_{\mathrm{n}}(k), \tag{1}$$

where $\mathbf{x}_{\mathrm{s}}(k)$, $\mathbf{x}_l(k)$, and $\mathbf{x}_{\mathrm{n}}(k) \in \mathcal{C}^{M\times 1}$ are statistically independent components representing the SOI, interferences, and sensor noise, respectively. $\mathbf{s}(k) = [s_1(k), \cdots, s_L(k)]^T \in \mathcal{C}^{L\times 1}$ is the signal waveform vector where $(\cdot)^T$ denotes the transpose and $\mathbf{a}_l = \mathbf{a}(\theta_l) \in \mathcal{C}^{M\times 1}$ is the SV associated with the lth source signal.

$\mathbf{x}_n(k)$ is assumed to be complex Gaussian noise vector with zero mean and covariance $\sigma_n^2\mathbf{I}_M$, and $\mathbf{I}_M$ stands for the $M \times M$ identity matrix. Assuming that the SV $\mathbf{a}(\theta_1)$ is known, then for a given beamformer weight vector $\mathbf{w}$, the beamformer performance is measured by the output signal-to-interference-plus-noise ratio (SINR) as follows

$$\mathrm{SINR} = \sigma_1^2|\mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta_1)|^2/\mathbf{w}^{\mathrm{H}}\mathbf{R}_{\mathrm{i+n}}\mathbf{w}, \tag{2}$$

where σ_1^2 and $\mathbf{R}_{\mathrm{i+n}}$ are the power of the DS and the INC matrix, respectively, and $(\cdot)^{\mathrm{H}}$ stands for Hermitian transpose. Assuming that the interfering signals are independent, the covariance matrix of the received signal vector is given by

$$\mathbf{R} = \sigma_1^2\mathbf{a}(\theta_1)\mathbf{a}^{\mathrm{H}}(\theta_1) + \sum_{l=2}^{L}\sigma_l^2\mathbf{a}(\theta_l)\mathbf{a}^{\mathrm{H}}(\theta_l) + \sigma_n^2\mathbf{I}, \tag{3}$$

where σ_n^2 and σ_l^2 represent the power of white Gaussian noise and of each interference component, respectively. The problem of maximizing the SINR in (2) can be cast as the following optimization problem:

$$\min_{\mathbf{w}} \mathbf{w}^{\mathrm{H}}\mathbf{R}_{\mathrm{i+n}}\,\mathbf{w} \quad \textbf{s.t.} \quad \mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta_1) = 1. \tag{4}$$

The solution to (4) yields the optimal beamformer given by

$$\mathbf{w}_{\mathrm{opt}} = \mathbf{R}_{\mathrm{i+n}}^{-1}\mathbf{a}(\theta_1)/\mathbf{a}^{\mathrm{H}}(\theta_1)\mathbf{R}_{\mathrm{i+n}}^{-1}\mathbf{a}(\theta_1). \tag{5}$$

Moreover, the array covariance matrix $\mathbf{R} = \mathrm{E}\{\mathbf{x}(k)\mathbf{x}^{\mathrm{H}}(k)\}$ is also given by [22]

$$\mathbf{R} = \mathbf{R}_{\mathrm{i+n}} + \mathbf{R}_{\mathrm{s}} = \int_{\Phi} P(\theta)\mathbf{a}(\theta)\mathbf{a}^{\mathrm{H}}(\theta)d\theta, \tag{6}$$

where $P(\theta)$ is the angular power spectrum of the signals, $\Phi = [\bar{\Theta} \cup \Theta_s]$ covers the union of the angular sectors of the DS and the interference-plus-noise signal, $\bar{\Theta} = \bar{\Theta}_l \cup \bar{\Theta}_n$ ($\bar{\Theta}_l$ is the interference angular sector and $\bar{\Theta}_n$ denotes the region excluding the signal and interference) and of the DS region, Θ_s (obtained through low-resolution direction finding methods [1]), while $\mathbf{R}_{\mathrm{s}} = \sigma_1^2\mathbf{a}_1\mathbf{a}_1^{\mathrm{H}}$ is the theoretical DS covariance matrix. Since the exact INC matrix, $\mathbf{R}_{\mathbf{i+n}}$ is unavailable, it is replaced by the SCM, $\hat{\mathbf{R}} = (1/K)\sum_{t=1}^{K}\mathbf{x}(k)\mathbf{x}^{\mathrm{H}}(k)$, where K is the number of snapshots.

3. PROPOSED INCPMP-SSM ALGORITHM

3.1. The INC matrix reconstruction

We estimate the corresponding parameters of the INC matrix according to its definition as in (3), namely the SVs, powers of all interferences and noise variance. Using low-resolution direction finding methods [23] to estimate the directions-of-arrival (DoA) of all interferences would lead to certain look direction estimation errors. That is to say, the DoAs of all interferences always lie in some angular sectors and $\bar{\Theta}_l$, $l = 2, 3, \cdots, L$ is assumed as the angular sector in which the lth interference is located.

The essence of the proposed method to reconstruct the INC matrix is according to the approach in [19]. Thus, we develop an idea which is based on the use of the maximum entropy power spectrum distribution over all possible directions and coarse estimates of the angular regions where the interference and noise lie:

$$\hat{P}(\theta) = \frac{1}{\epsilon_p|\mathbf{a}^{\mathrm{H}}(\theta)\hat{\mathbf{R}}^{-1}\mathbf{b}_1|^2}, \tag{7}$$

where $\mathbf{b}_1 = [\,1\ 0\ \cdots\ 0\,]^{\mathrm{T}}$, $\epsilon_p = 1/\mathbf{b}_1^{\mathrm{T}}\hat{\mathbf{R}}^{-1}\mathbf{b}_1$. Utilizing (6), restricted to the angular sector $\bar{\Theta}$ and using the maximum entropy power estimate (7), the INC matrix can be reconstructed by numerically evaluating

$$\hat{\mathbf{R}}_{\mathrm{i+n}} = \int_{\bar{\Theta}} \hat{P}(\theta)\mathbf{a}(\theta)\mathbf{a}^{\mathrm{H}}(\theta)d\theta. \tag{8}$$

Sampling $\bar{\Theta}$ uniformly with $Q \gg M$ sampling points spaced by $\Delta\theta$, (8) can be approximated by

$$\hat{\mathbf{R}}_{\mathrm{i+n}} \approx \sum_{i=1}^{Q}\frac{\mathbf{a}(\theta_i)\mathbf{a}^{\mathrm{H}}(\theta_i)}{\epsilon_p|\mathbf{a}^{\mathrm{H}}(\theta_i)\hat{\mathbf{R}}^{-1}\mathbf{b}_1|^2}\Delta\theta, \tag{9}$$

which needs more computations (multiplications and summations). However, in the proposed method, instead of summation over the whole angular sector $\bar{\Theta}$ (requiring Q sampling points) away from the SOI, we only make summation over the small angular sector $\bar{\Theta}_l$ (thus requiring less sampling points) of the lth interference to obtain the lth interference covariance matrix as

$$\hat{\mathbf{C}}_l \approx \sum_{lj=1}^{J_l}\frac{\mathbf{a}(\theta_{lj})\mathbf{a}^{\mathrm{H}}(\theta_{lj})}{\epsilon_p|\mathbf{a}^{\mathrm{H}}(\theta_{lj})\hat{\mathbf{R}}^{-1}\mathbf{b}_1|^2}\Delta\theta, \tag{10}$$

where $\mathbf{a}(\theta_{lj})$ is the SV associated with $\{\theta_{lj} \in \bar{\Theta}_{ld}\}_{j=1}^{J_l}$, $\bar{\Theta}_{ld}$ is a discretization of the angular sector $\bar{\Theta}_l$ with $J_l \ll Q$ sampling points. This leads to tiny angular interval. Since only one interference signal is assumed in the uncertainty region ($\bar{\Theta}_l$), the principal eigenvector of the constructed matrix will be the SV we are looking for. To avoid the EVD processing, and reduce the complexity, the power method [21] is employed to estimate the principal eigenvalue and the corresponding eigenvector of the reconstructed INC, $\hat{\mathbf{C}}_l$, based on the theorem below:

Theorem 1: A Hermitian matrix $\mathbf{A} \in \mathcal{C}^{N\times N}$ has N orthogonal eigenvectors $\mathbf{u}_1, \cdots, \mathbf{u}_N$ ($\|\mathbf{u}_i\|_2 = 1$ for $i \in [1, \cdots, N]$). Assume its eigenvalues satisfy the relation $|\lambda_1| > |\lambda_2| \geq \cdots \geq |\lambda_N|$, and let $\mathbf{v}_0 = \sum_{i=1}^{N}\alpha_i\mathbf{u}_i$ ($\alpha_i \neq 0$). Take the vector $\mathbf{v}_0$ as the initial vector, and form a vector sequence according to the power of $\mathbf{A}$ as

$$\mathbf{d}_k = \begin{cases}\mathbf{v}_k = \mathbf{A}\mathbf{v}_{k-1}\\ m_k = \|\mathbf{v}_k\|_\infty\\ \bar{\mathbf{v}}_k = \mathbf{v}_k/m_k, \quad (k = 1, 2, \cdots)\end{cases} \tag{11}$$

then it holds that [24]

$$\lim_{k\longrightarrow+\infty}\bar{\mathbf{v}}_k = \frac{\mathbf{u}_1}{\|\mathbf{u}_1\|_\infty}. \tag{12}$$

According to this power method theorem, the principal eigenvalue and corresponding eigenvector of matrix $\mathbf{A}$ can be computed. Utilizing this theorem, we intend to find the largest eigenvalue and the principal eigenvector of the reconstructed interference matrices that retains as much as possible the power and SV of the interferences. By applying theorem 1 to the reconstructed matrix $\hat{\mathbf{C}}_l$ for every interference, the power of the interference and the steering vector are obtained. Therefore we obtain the lth interference covariance matrix term $\hat{\sigma}_l^2\hat{\mathbf{a}}_l\hat{\mathbf{a}}_l^{\mathrm{H}}$, the SVs $\hat{\mathbf{a}}_l$ and their powers $\hat{\sigma}_l^2$, ($l = 2, 3, \cdots, L$). Since each region $\bar{\Theta}_l$ is small, $\hat{\mathbf{C}}_l$ will be approximately a rank-one matrix, and thus the convergence in Theorem 1 should be fast. Finally we obtain the interference covariance matrix $\sum_{l=2}^{L}\hat{\sigma}_l^2\hat{\mathbf{a}}_l\hat{\mathbf{a}}_l^{\mathrm{H}}$. In the noise region $\bar{\Theta}_n$, which is the complement of $\bar{\Theta}_l \cup \bar{\Theta}_s$, we employ the maximum entropy power spectrum distribution in (7) to calculate the noise power as the average

$$\bar{\sigma}_n^2 = \frac{1}{T}\sum_{t=1}^{T}\frac{1}{\epsilon_p|\mathbf{a}^{\mathrm{H}}(\theta_t)\hat{\mathbf{R}}^{-1}\mathbf{b}_1|^2}, \tag{13}$$

where θ_t is a discrete sample point in $\bar{\Theta}_n$, T is the number of sample points. This approximation can be justified if we assume that $\mathbf{x}(k)$ is comprised of only complex Gaussian white noise, that is to say, the data covariance matrix becomes $\hat{\mathbf{R}} = \sigma_n^2\mathbf{I}$. In this case, based on (7) the noise power can be obtained as follows:

$$\hat{P}(\theta) = \frac{1}{\epsilon_p\left|\mathbf{a}^{\mathrm{H}}(\theta)\left(\sigma_n^2\mathbf{I}\right)^{-1}\mathbf{b}_1\right|^2} = \frac{(\sigma_n^2)^2}{\epsilon_p}, \tag{14}$$

where $\epsilon_p = \frac{1}{\mathbf{b}_1^{\mathrm{T}}\hat{\mathbf{R}}^{-1}\mathbf{b}_1} = \sigma_n^2$. By replacing ϵ_p into (14) we can write

$$\hat{P}(\theta) = \sigma_n^2. \tag{15}$$

(15) implies that in the presence of only noise components, the power is composed of residual noise components which is same as the actual noise. Then, from (13) and (15) we can conclude that the actual noise power is estimated as $\hat{\sigma}_n^2 \approx \bar{\sigma}_n^2$.

3.2. Desired signal steering vector estimation

Assume that the DS lies in the angular sector Θ_s which is assumed to be distinguishable from the location of the interference signal. In the proposed INCPMP-SSM method, the maximum entropy power spectrum calculated by the reconstructed signal plus-noise covariance matrix, $\hat{\mathbf{R}}_{\text{s+n}}$ is denoted as $\hat{\mathbf{P}}_{\text{s+n}}(\theta)$ and that computed by the SCM, $\hat{\mathbf{R}}$ is depicted by $\mathbf{P}(\theta)$ are required to be matched in the angular sector Θ_s. Besides, the spectrum corresponding to $\hat{\mathbf{R}}_{\text{s+n}}$ should approximate to the average noise power $\hat{\sigma}_n^2$ in the complement angular sector of Θ_s, which is denoted as $\bar{\Theta}$.

The spectrum matching processing for signal-plus noise covariance matrix reconstruction has two main objectives: (i) minimize the difference between $\hat{\mathbf{P}}_{\text{s+n}}(\theta)$ and $\mathbf{P}(\theta)$ in the angular sector of Θ_s. (ii) constrain the difference between the average noise power $\hat{\sigma}_n^2$ and the spatial spectrum of $\hat{\mathbf{P}}_{\text{s+n}}(\theta)$ in the angular sector $\bar{\Theta}$.

Since a covariance matrix is always positive semidefinite, the positive semidefinite requirement of the reconstructed signal-plus-noise covariance matrix is guaranteed. The proposed INCPMP-SSM algorithm to obtain $\hat{\mathbf{R}}_{\text{s+n}}$ can be formulated as the optimization problem:

$$\begin{aligned}&\min_{\hat{\mathbf{R}}_{\text{s+n}}}\|\hat{\mathbf{P}}_{\text{s+n}}(\theta) - \mathbf{P}(\theta)\|_2\\ &\text{s.t.}\|\hat{\mathbf{P}}_{\text{s+n}}(\theta) - \hat{\sigma}_n^2\|_2 < \zeta\\ &\hat{\mathbf{R}}_{\text{s+n}} \in \mathbf{S}_+^M\end{aligned} \tag{16}$$

where ζ is a relatively small value, and $\mathbf{S}_+^M$ is a set of $M \times M$ positive semidefinite matrices. The above expression is rewritten as

$$\begin{aligned}&\min_{\hat{\mathbf{R}}_{\text{s+n}}}\Big(\int_{\Theta_s}\Big|\frac{1}{\epsilon_p\left|\mathbf{a}^{\mathrm{H}}(\theta)\hat{\mathbf{R}}_{\text{s+n}}^{-1}\mathbf{b}_1\right|^2} - \frac{1}{\mathbf{a}^{\mathrm{H}}(\theta)\hat{\mathbf{R}}^{-1}\mathbf{a}(\theta)}\Big|^2 d\theta\Big)^{1/2}\\ &\text{s.t.}\int_{\bar{\Theta}}\Big(\Big|\frac{1}{\epsilon_p\left|\mathbf{a}^{\mathrm{H}}(\theta)\hat{\mathbf{R}}_{\text{s+n}}^{-1}\mathbf{b}_1\right|^2} - \hat{\sigma}_n^2\Big|^2 d\theta\Big)^{1/2} < \zeta\\ &\hat{\mathbf{R}}_{\text{s+n}} \in \mathbf{S}_+^M\end{aligned} \tag{17}$$

To simplify the calculation of (17), we choose a finite number of angles $\nu_j \in \Theta_s(j = 1, 2, \cdots, G)$ and $\phi_i \in \bar{\Theta}(i = 1, 2, \cdots, Q)$ to discretize the angular sector Θ_s and the complement angular sector $\bar{\Theta}$, respectively. Hence, (17) becomes

$$\begin{aligned}&\min_{\hat{\mathbf{R}}_{\text{s+n}}}\left\|\begin{pmatrix}\frac{1}{\epsilon_p\left|\mathbf{a}^{\mathrm{H}}(\nu_1)\hat{\mathbf{R}}_{\text{s+n}}^{-1}\mathbf{b}_1\right|^2}\\ \vdots\\ \frac{1}{\epsilon_p\left|\mathbf{a}^{\mathrm{H}}(\nu_G)\hat{\mathbf{R}}_{\text{s+n}}^{-1}\mathbf{b}_1\right|^2}\end{pmatrix} - \begin{pmatrix}\frac{1}{\mathbf{a}^{\mathrm{H}}(\nu_1)\hat{\mathbf{R}}^{-1}\mathbf{a}(\nu_1)}\\ \vdots\\ \frac{1}{\mathbf{a}^{\mathrm{H}}(\nu_G)\hat{\mathbf{R}}^{-1}\mathbf{a}(\nu_G)}\end{pmatrix}\right\|_2\\ &\text{s.t.}\left\|\begin{pmatrix}\frac{1}{\epsilon_p\left|\mathbf{a}^{\mathrm{H}}(\phi_1)\hat{\mathbf{R}}_{\text{s+n}}^{-1}\mathbf{b}_1\right|^2}\\ \vdots\\ \frac{1}{\epsilon_p\left|\mathbf{a}^{\mathrm{H}}(\phi_Q)\hat{\mathbf{R}}_{\text{s+n}}^{-1}\mathbf{b}_1\right|^2}\end{pmatrix} - \mathbf{1}.\hat{\sigma}_n^2\right\|_2 < \zeta\\ &\hat{\mathbf{R}}_{\text{s+n}} \in \mathbf{S}_+^M\end{aligned} \tag{18}$$

where $\mathbf{1}$ represents the $Q \times 1$ vector with all elements equal to one. However, (18) is a nonconvex optimization problem since the objective function and the inequality constraint function are nonconvex functions. To convert (18) to a convex optimization problem, we first use the reciprocal of the spatial spectrum to replace the original spatial spectrum, and then define $\mathbf{V}_s = [\mathbf{a}(\nu_1), \mathbf{a}(\nu_2), \cdots, \mathbf{a}(\nu_S)]$, $\mathbf{Y}_s = [\mathbf{a}(\phi_1), \mathbf{a}(\phi_2), \cdots, \mathbf{a}(\phi_Q)]$, and $\mathbf{D}_s = \hat{\mathbf{R}}_{\text{s+n}}^{-1}$. Consequently, a new optimization problem is obtained as follows:

$$\begin{aligned}&\min_{\mathbf{D}_s}\left\|\text{diag}(\mathbf{V}_s^{\mathrm{H}}\mathbf{D}_s\mathbf{V}_s) - \text{diag}(\mathbf{V}_s^{\mathrm{H}}\hat{\mathbf{R}}^{-1}\mathbf{V}_s)\right\|_2\\ &\text{s.t.}\left\|\text{diag}(\mathbf{Y}_s^{\mathrm{H}}\mathbf{D}_s\mathbf{Y}_s) - \mathbf{1}.(1/\hat{\sigma}_n^2)\right\|_2 < \zeta\\ &\mathbf{D}_s \in \mathbf{S}_+^M\end{aligned} \tag{19}$$

where $\text{diag}(\cdot)$ is an operator which returns the diagonal vector of a matrix, ζ is a relatively small value which guarantees that the spatial spectrum of $\text{diag}(\mathbf{V}_s^{\mathrm{H}}\mathbf{D}_s\mathbf{V}_s)$ is close to the average noise power $\hat{\sigma}_n^2$ in the angular sector. We can find that (19) is a convex optimization problem, and thus can be efficiently solved by CVX [25]. By solving this optimization problem, the reconstructed signal-plus-noise covariance matrix $\mathbf{D}_s = \hat{\mathbf{R}}_{\text{s+n}}^{-1}$ is achieved. However, the solution of (19) is the reconstructed signal plus-noise covariance matrix but not the covariance matrix of DS. Therefore, we have to remove the noise components from $\mathbf{D}_s = \hat{\mathbf{R}}_{\text{s+n}}^{-1}$. Recall that $\hat{\sigma}_n^2$ is the average noise power and assuming that the sensor noise is spatially white Gaussian noise, then the noise covariance matrix can be estimated by $\hat{\mathbf{R}}_{\text{n}} = \hat{\sigma}_n^2\mathbf{I}$. Hence, the covariance matrix of DS is calculated by

$$\hat{\mathbf{R}}_{\text{s}} = \mathbf{D}_s^{-1} - \hat{\sigma}_n^2\mathbf{I}. \tag{20}$$

According to the summary of the power method, (11), the principal eigenvalue $\hat{\sigma}_1^2$ and the eigenvector $\hat{\mathbf{a}}(\theta_1)$ of $\hat{\mathbf{R}}_{\text{s}}$ can be computed. Note that the proposed SV estimation algorithm requires little prior information, such as imprecise knowledge of the array geometry and angular sectors, while knowledge of the assumed SV is not essential.

4. SIMULATIONS

In this section, a ULA with $M = 10$ omnidirectional sensors is used. It is assumed that there is one DS from the presumed direction $\bar{\theta}_1 = 10^\circ$ while the uncorrelated interference signals are impinging from 30° and 50°. The input interference to noise ratios (INRs) of the two interferers are both set to 30 dB. The noise is

Algorithm 1 Proposed PMP-SSM Adaptive Beamforming

Input: Array received data vector $\{\mathbf{x}(k)\}_{k=1}^{K}$,
Initialize: $m_0 = 1$, $\mathbf{u}_0 = [1, 1, \cdots, 1]^T$, δ;
Compute $\hat{\mathbf{R}} = (1/K)\sum_{k=1}^{K} \mathbf{x}(k)\mathbf{x}^H(k)$;
For $l = 2 : L$
  Construct $\hat{\mathbf{C}}_l$ using (10),
  Apply power method theorem in (11) to $\hat{\mathbf{C}}_l$,
  Obtain $\hat{\sigma}_l^2$ and $\hat{\mathbf{a}}_l$,
End For
Compute estimated noise power using (13);
Construct INC matrix as $\hat{\mathbf{R}}_{\mathrm{i+n}} = \sum_{l=2}^{L} \hat{\sigma}_l^2 \hat{\mathbf{a}}_l \hat{\mathbf{a}}_l^{\mathrm{H}} + \hat{\sigma}_n^2 \mathbf{I}$.
Compute $\mathbf{D}_s = \hat{\mathbf{R}}_{\mathrm{s+n}}^{-1}$ using (19);
Estimate the desired signal covariance matrix,$\hat{\mathbf{R}}_{\mathrm{s}}$ utilizing (20),
Apply power method theorem in (11) to $\hat{\mathbf{R}}_{\mathrm{s}}$,
Obtain $\hat{\sigma}_1^2$ and $\hat{\mathbf{a}}_1$.
Design proposed beamformer using (5),
Output: Proposed beamforming weight vector $\mathbf{w}_{\mathrm{prop}}$.

regarded as complex Gaussian temporally and spatially white process with zero mean and unit variance. The proposed (INCPMP-SSM) method is compared with the beamformer in [15] (INC-SUB), the reconstruction-estimation based beamformer in [8] (INC-EST), the beamformer in [18] (INC-OS), the beamformer in [13] (INC-SPSS), the beamformer in [26] (INC-SV), the beamformer in [19] (INC-MEPS) and the beamformer in [27] (INC-FADL). In the INC-SV and INC-EST beamformers the number of sampling points for interference-plus-noise region is fixed at 200. In the beamformer INC-SV, the upper bound of the norm of the SV mismatch is set to $\sqrt{0.1}$. In the proposed INCPMP-SSM method, $k = 4$ iterations are used to compute the dominant eigenvector while we perform 100 Monte-Carlo runs. The dominant eigenvectors is fixed 7 in INC-OS. The energy percentage ρ set as 0.9 in INC-SUB. The angular sector of the DS is set to be $\Theta_s = [\bar{\theta}_1 - 5^\circ, \bar{\theta}_1 + 5^\circ]$ where the interference angular sector is $\bar{\Theta} = [-90^\circ, \bar{\theta}_1 - 5^\circ) \cup (\bar{\theta}_1 + 5^\circ, 90^\circ]$.

In the example, we evaluate INCPMP-SSM in the presence of look direction and model mismatches due to the sensor displacement errors. In this example, it is assumed that the DS and the interferers are uniformly distributed in $[-5^\circ, 5^\circ]$ while the difference between the actual and assumed SV is modeled as array geometry errors, assuming the sensor position is drawn uniformly from $[-0.05, 0.05]$ wavelength and the DoA of the DS and the actual sensor position changes from run to run while remaining constant over samples.

In Fig. 1, we compare the SINR performance versus the SNR where the number of snapshots is fixed at $K = 50$. It is well-known that the theoretical INC is a linear combination of the SVs and powers of interferences, and the reconstructed INC in proposed method (INCPMP-SSM) is the linear combination of estimated powers $\hat{\sigma}_l^2$ and corresponding SVs $\hat{\mathbf{a}}_l$. It is demonstrated that the proposed INCPMP-SSM method outperforms the INC-MEPS and INC-SUB and it is better than INC-EST. From the results, it is observed that, because of random sensor position errors, there is an almost constant performance loss for INC-OS and INC-SPSS regardless of the input SNR. At SNRs higher than 0 dB, the INC-SV beamformer has a performance loss because of the look direction mismatch. On the other hand, the proposed INCPMP-SSM beamformer almost attains the optimal output SINR under these mismatches for all SNRs. It should be noted that, since the INC-FADL beamformer utilizes the assumed SV to compute the weight vector, its performance for low SNRs is better than the other beamformers.

In Fig. 2, all tested beamformers are evaluated as the number of snapshots is increased at SNR=10 dB. The performance of the INCPMP-SSM method stems from its highly accurate estimate of the INC matrix with respect to the theoretical one, which enhances the robustness of the proposed INC against random look direction and array geometry errors over the snapshots. The results demonstrate that the number of snapshots does not significantly affect the output SINR of tested beamformers, and the INC-MEPS and INC-SUB beamformers almost get the same performance as the INCPMP-SSM method.

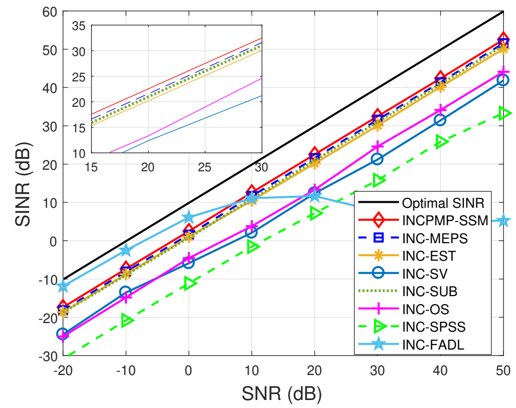

Fig. 1: Output SINR versus input SNR

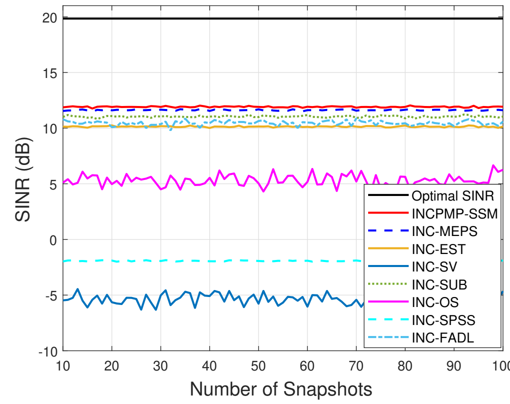

Fig. 2: Output SINR versus number of snapshots

5. CONCLUSION

In this work, an efficient and accurate estimation of the INC matrix and DS steering vector has been proposed, where the eigenvalues and eigenvectors lying within the interval of the SOI and the interference angular regions are computed by the power method that estimates the actual power and SV of the interferences and desired signal. Furthermore, an efficient algorithm based on the matching spectrum is devised to reconstruct the DS covariance matrix and estimate the SV of the SOI. Simulation results have shown that the proposed INCPMP-SSM algorithm outperforms recently reported approaches.

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